Aerosol Deposition Measurements as a Function of Reynolds Number for Turbulent Flow in a Ninety-Degree Pipe Bend

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NOTICE: This is an Accepted Manuscript of an article published in Aerosol Science & Technology on January 3, 2011, available online: http://www.tandfonline.com/10.1080/02786826.2010.538092.

This article should be cited as:

Keywords: Aerosol Deposition, turbulent deposition, Bend Flow, Eddy-Interaction Model

Abstract

An experimental and numerical investigation of the effect of the Reynolds number (Re) on the deposition of aerosol particles in a 90° pipe bend for turbulent flow was performed. Deposition fraction data were measured for a range of Stokes numbers (Stk) at different flow Re (10 250, 20 500 and 30 750) higher than those of most previous studies where Re was \( \leq 10 000 \). The data show good agreement with previous studies for Stk > 0.4, demonstrating that increased Re does not significantly alter the trend of deposition fraction with Stokes number (Stk) in this range. However, a noticeable increase in deposition was detected for \( 0.1 \leq \text{Stk} \leq 0.4 \). At Stk=0.15, an increase in Re from 10 250 to 30 750 caused a factor of 2.6 increase in deposition fraction from 0.14 to 0.36. Numerical simulations were completed, using the Reynolds Averaged Navier-Stokes (RANS) equations with the Shear Stress Transport turbulence model. Modeling with inertial impaction only (i.e. neglecting turbulent dispersion), the results accurately
reproduced the general trends seen in the experimental data; however, they failed to detect the Re effect at low Stk seen experimentally. The inclusion of turbulent particle tracking in the RANS simulation via an eddy interaction model did not improve the results. However, an analytical analysis of the particle tracking equation drawing data from the numerical results, showed that the experimentally observed effect of Re at low Stk can be attributed to damped particle response to velocity fluctuations at the eddy frequency scale.

**Introduction**

There are many practical examples of aerosols flowing through bends, which create challenging engineering problems. These include: the prevention of icing in curved aircraft intake ducts (which motivated the study of Hacker et al., 1953), the removal of particulates from industrial duct bends (a concern of Peters and Leith, 2004), extractive sampling of contaminants in gas streams (which motivated the studies of Pui et al., 1987 and McFarland et al., 1997), the erosion of pipe bends due to solid aerosol particles (a concern of Yeung 1979), and the deposition of particles in the respiratory tract (for example the throat model of Zhang et al., 2004). Results are commonly described in terms of the deposition fraction (DF), which is defined as the fraction of the aerosol particles flowing into the bend that deposit on the pipe walls.

Aerosol deposition in bends has been studied experimentally, analytically, and numerically for both the laminar case (Re < 2300) and the turbulent case (Re > 2300). Experimental results for the laminar case have been obtained by Johnston and Muir (1973), Johnston et al. (1977), Pui et al. (1987), and Sato et al. (2003). These results indicated a non-linear increase of the deposition fraction with increased Stokes number, and a possible increase of the deposition fraction with increased flow Reynolds number. Theoretical calculations for the laminar case have been made by Landahl and Hermann (1949), Hacker et al. (1953), Yeh (1974), Cheng and Wang (1975), Crane and Evans (1977), Cheng and Wang (1981), and Tsai and Pui (1990). Based on the most rigorous results (Cheng and Wang, 1981, and Tsai and Pui, 1990), a decrease of the curvature ratio (the ratio of the bend radius to the inlet tube radius) and an increase of the Reynolds numbers both cause increase of DF; however, there is disagreement among some of the cited authors with these conclusions.

Fewer researchers have obtained experimental data for turbulent flows (e.g. Pui et al., 1987, McFarland et al., 1997, and Peters and Leith, 2004). Data from the literature for a comparable range of experiments has been collected and reproduced in Figure 1. McFarland et al. (1997) considered effects of bend curvature, and found that curvature ratios less than 4 or 5 lead to significantly increased deposition. Pui et al. (1987) considered different Reynolds numbers (Re < 10 000) and showed no significant differences in DF.
Figure 1: Comparison of data from previous studies of aerosol deposition in turbulent flow through a 90° bend (including the current Re = 10 250 case)

Numerical deposition studies for turbulent bend flows have been made by McFarland et al. (1997), varying the Reynolds number from 3200 to 19 800 (and the Stokes number up to 0.67). Their results showed that the effect of the Reynolds number on the deposition fraction is insignificant, changing by less than 5% at a given Stokes number. Also, an increase of the curvature ratio led to a decrease of the deposition fraction. Breuer et al. (2006) simulated an experimental case of Pui et al. (1987), Re=10 000, \( R_o=R_b/a=5.7 \) (curvature ratio), using a large eddy simulation model. The flow into the bend was fully developed. Their results are shown in Figure 1 and agree very closely with the experimental data of Pui et al. (1987), who reported “estimated” uncertainties in the experimentally measured Stokes number and DF of 5% and 3%, respectively. Breuer et al. (2006) used a Lagrangian particle tracking scheme to determine the particle trajectories, with a drag coefficient dependant on the particle Reynolds number, and the influence of the particles on the flow was considered negligible (called one-way coupling). McFarland et al. (1997) used the RANS equations with the Reynolds Stress turbulence model. To calculate the instantaneous particle velocity to be used in the particle equation of motion, the turbulent velocity fluctuations were modeled with an eddy interaction model.

In summary, past studies for aerosol deposition in turbulent flow suggest that the influence of the Reynolds number is insignificant (although most studies considered Re ≤ 10000), but that the influence of curvature ratio (for values below 4 or 5) and the Stokes number are significant. In the present work, experimental data are presented for flow Reynolds numbers of 10 250, 20 500 and 30 750 (extending to a higher range than previous studies) and Stokes numbers
between 0.1 and 1.0. This allows the data from the previous studies to be extended up to a Reynolds number of 30,750. In the course of this experimental investigation, a measurable Reynolds number effect for small Stokes numbers was found. Numerical simulations were added to see if they could capture or explain this effect. Finally an analytical model, based on particle response to turbulent eddies, was developed to further explore the influence of turbulent fluctuations on low Stokes number particles seen in the experimental data.

**Parameters**

Four main mechanisms are responsible for the deposition of the particles on the wall of a 90° bend in a smooth-walled, circular cross-section pipe: gravitational settling, molecular diffusion, turbulent dispersion (i.e. dispersion of particles caused by turbulent eddies), and mean-flow induced inertial impaction (which occurs when particles have too much centrifugal inertia to pass through the bend). Assuming aerosol particles stick to the pipe wall upon contact, then the deposition fraction for a circular cross-section 90° bend depends on the geometry of the test section (pipe internal radius \(a\), bend radius \(R_b\), and gravitational constant \(g\)), the characteristics of the air flow (volumetric flow rate \(Q\), viscosity \(\mu\), and density \(\rho\)), the characteristics of the flow of aerosol particles (the volumetric flow rate of particles into the bend \(Q_{in}\), the volumetric flow rate of particles out of the bend \(Q_{out}\), particle diameter \(d_p\), particle density \(\rho_p\)), and the diffusion coefficient of the aerosol particles in air \(D\). Thus, DF is a function of 11 different parameters: \(Q_{in}, Q_{out}, a, R_b, Q, \rho, \mu, d_p, \rho_p, D,\) and \(g\). However, through dimensional analysis using the Buckingham Pi theorem, the functional dependence can be simplified in terms of eight independent non-dimensional groupings:

\[
DF = f\left(\frac{Q_{in}}{Q}, \frac{Q_{out}}{Q}, R_b, Re, Stk, \xi, Pe, Ri\right)
\]

(1)

where:

- \(\frac{Q_{in}}{Q}\) is the volume fraction of particles entering the bend;
- \(\frac{Q_{out}}{Q}\) is the volume fraction of particles leaving the bend;
- \(R_b = \frac{R_b}{a}\) is the curvature ratio, \(R_b\) is the bend radius, and \(a\) is the pipe radius;
- \(Re = \frac{2Q\rho}{\pi a \mu}\) is the flow Reynolds number based on the pipe diameter and average flow
velocity \( (Q = \pi a^2 V) \);

\[
Stk = \frac{C_e \rho_d d_p^2 Q}{18 \pi \mu a^3}
\]

is the Stokes number;

\( C_e \) is the Cunningham slip correction factor which may be calculated (Hinds, 1999) based on the particle diameter and the mean free path \( (\lambda) \) of the carrier gas as

\[
C_e = 1 + 2.52 \left( \frac{\lambda}{d_p} \right);
\]

\( \xi = \frac{\rho_d}{\rho} \) is the density ratio;

\[
Pe = \frac{\mu}{D \rho}
\]

is the Peclet number; and

\[
Ri = \frac{\pi^2 g a^5}{Q^2}
\]

is the Richardson number which is equivalent to the square root of the inverse of the Froude number, \( \text{Fr} = \frac{Q}{\sqrt{\pi^2 g a^5}} \).

If the volume concentration of particles in the flow is very small \((\leq 0.0001\%, \text{Elghobashi, 1994})\), one-way coupling can be assumed in which the motion of particles is affected by the air flow but the air flow is not affected by the presence of the particles. In the present investigation, the volumetric concentration of particles is of the order of 0.0001%. Thus, it is reasonable to assume that the flow is not influenced by the particles and that particle interactions are negligible, so that molecular diffusion is the only way for the volume fraction of the particles to influence the deposition fraction. The Peclet number is the inverse of the non-dimensional diffusion coefficient, and is very large for the flows under consideration \((Pe > 6.25 \times 10^5, \text{Wilson, 2008})\), so molecular diffusion can be ignored. This means that the two volume fractions and the Peclet number can be removed from the right hand side of Eq. (1). Furthermore, the Richardson number represents the magnitude of the influence of gravity on the flow-field and on the particles in the flow-field, and is very small relative to other terms \((Ri < 2.1 \times 10^{-4}, \text{Wilson, 2008})\), so gravitational effects can also be ignored. Thus Eq. (1) can be reduced to:

\[
\text{DF} = f\left(R_o, \text{Re}, \text{Stk}, \xi\right).
\]

Cheng and Wang (1981) reported that the deposition fraction depends on \( \xi, \text{Re}, \text{Stk}, \) the freestream particle Reynolds number \( (\text{Re}_p, \infty = \frac{V \mu d_p}{\mu}) \), and the inception parameter \( \frac{d_p}{2a} \).
However, it is easy to reduce their conclusion to the one given here. Since the following is true

\[
\frac{d_p}{2a} = \frac{\text{Re}_{p,c}}{\text{Re}} = 3 \sqrt{\frac{\text{Stk}}{\xi \text{Re}}},
\]

the freestream particle Reynolds number, and the interception parameter \( \frac{d_p}{2a} \), can be written as functions of the other parameters. The importance of Equation (2) is that DF results for fixed \( R_0 \) and \( \xi \) can be plotted solely as a function of \( \text{Re} \) and \( \text{Stk} \) as is done below.

**Experimental Method**

Deposition measurements were performed using an ultra-violet spectroscopy measurement technique adapted from a previous study of DeHaan and Finlay (2001) with vitamin E (alpha-tocopheryl acetate) as the aerosol particle material. Figure 2 shows a schematic of the apparatus used.

![Figure 2: Schematic of Experimental Apparatus (Not To Scale)](image)

The bend was positioned so that the aerosol flowed vertically downward into the test section. The flow Reynolds number is referenced to the inner diameter of the pipe (10.2 mm). An entrance length of 27 diameters ensured that the turbulent flow entering the test section was fully developed. The bend geometry is described by the curvature ratio \( R_0 = R_b/a = 7.4\pm0.5 \). The bend was created from standard stainless steel (grade 304) tubing using a standard tube bender. To minimize pinching of the cross-section, the tubing was filled with sand prior to bending.
This pinching was quantified by the fractional reduction of the tube diameter in the pinched direction, which was about 5%. According to the experimental investigation of McFarland et al. (1997) the effect is minor for pinching less than 25%, so no effect due to pinching is expected in this study. Downstream of the test section, there was a 3.8 cm straight section of tubing before the filter.

Aerosol particles consisting of liquid of vitamin E were generated by a Vibrating Orifice Aerosol Generator (Model 3450, TSI Inc., Shoreview, MN). Excess electrostatic charge was neutralized using an Aerosol Neutralizer (Model 3054, TSI Inc., Shoreview, MN). A Particle Size Distribution Analyzer (PSD, Model 3603, TSI Inc., Shoreview, MN) was used to measure the particle size (with a 2% bias limit) and to monitor the highly-monodisperse distribution. The aerosol flow was driven by a vacuum pump downstream, and the mass flow rate was measured with a thermal mass flow meter (Model 5863S, Brooks Instrument, Hatfield, PA).

A portion of the vitamin E droplets deposits on the bend wall as the aerosol laden flow passes through the bend. Since the vitamin E droplets are liquid and non-volatile, they stick to the wall and do not re-enter the flow. In a typical experiment, the aerosol flow was passed through the test section at steady state for 0.5 hours to ensure sufficient deposition for accurate measurement. Each experiment was repeated at least three times to allow calculation of the precision uncertainty. The deposited vitamin E was subsequently washed from the bend and from the filter (Marquest Respirgard-II 303 bacterial filters, Vital Signs Inc., Totowa, NJ) including the straight section of pipe immediately downstream of the bend, using the procedure outlined below. The amount of vitamin E deposited in each part of the apparatus was measured with a spectrophotometer (Model 8453, Agilent Technologies Inc., Santa Clara, CA), which enabled calculation of the deposition fraction. The deposition fraction is the ratio of the mass of particles that deposited on the bend wall to the total mass of particles, which flowed into the bend.

**Wash Procedure**

Following deposition, the entrance section, exit section, bend sections, and filter were separately washed with methanol to collect the vitamin E in solution. First the entrance section was cleaned by soaking it in acetone to remove and dispose of any vitamin E deposited upstream of the test section. The exit section was then soaked in a measured amount of methanol (typically 15 mL) to remove the vitamin E. This methanol/vitamin E solution from the exit section wash was then poured back and forth through the filter to collect additional Vitamin E that travelled through the bend and exit. Because the filter pad absorbed a significant portion of the total methanol used, to obtain sufficient mixed solution for sampling in the spectrophotometer, a portion of the methanol solution that was absorbed into the filter pad was removed by pipetting it with a disposable Pasteur pipette. This solution was identified as the filter solution, which could be used to quantify the amount of Vitamin E passing through the bend. This washing procedure was necessary because it was difficult to disconnect the filter from a robust filter holder, which was used to connect the filter to the test section at one end and
the hose to the vacuum pump at the other end. The 90° bend was subsequently washed with a similar measured amount of methanol by pouring it into the bend, washing it back and forth within the bend, and allowing it to soak for at least 5 minutes.

The spectral absorbance of each solution was measured in the spectrophotometer to determine the concentration of vitamin E in solution. Since the amount of methanol used in the wash procedure was known, the mass of vitamin E and hence the deposition fraction in the bend could be determined. The bias limits for the key parameters in the experiment are summarized in Table 1 below.

Table 1: Experimental results (Curvature ratio is 7.4 ± 0.5. Density ratio is 770 ± 20. The uncertainty in Stk is ±6%)

<table>
<thead>
<tr>
<th>Re and initial particle velocity [m/s]</th>
<th>Diameter of Particle [µm]</th>
<th>Stk</th>
<th>Deposition Fraction (DF)</th>
<th>Bias Limit</th>
<th>Precision</th>
<th>Uncertainty</th>
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<tr>
<td>10250±200 15.41</td>
<td>3.66</td>
<td>0.12</td>
<td>0.05</td>
<td>0.003</td>
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<tr>
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<td>0.32</td>
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<td>0.94</td>
<td>0.003</td>
<td>0.022</td>
<td>0.022</td>
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</table>
A detailed uncertainty analysis was performed on the full experimental procedure in accordance with the ANSI/ASME Measurement Uncertainty Standard (1985), as detailed by Wilson (2008). The uncertainty of the data consists of two parts: the precision uncertainty (apparent as scatter in the measurements caused by random error) and the propagated bias uncertainty (represented as the manufacturers limits on the ability of the equipment to measure true values). The propagated uncertainty from all sources was calculated and is reported using error bars on the figures below.

**Experimental Results**

The experimental results (along with a breakdown of the calculated uncertainties) are summarized in Table 1. Figure 3 shows a plot of the current results for $\text{Re} = 10,250$, $20,500$, and $30,750$, with two-dimensional error bars representing calculated total experimental uncertainties. In general, as the Stokes number is increased, there is a corresponding increase of deposition fraction. For all Re, at a Stokes number of 0.8 the deposition fraction is about 80%, and as the Stokes number is increased further, the rate of increase of the deposition fraction flattens out. Even though the curvature ratios are different by 30% (7.4 in the present study versus 5.7 for Pui et al.), as shown on Figure 1, the corresponding results for $\text{Re} = 10,000$ are almost identical. The experimental results of McFarland et al. (1997) show that the deposition fraction is expected to change by less than $+0.05$ going from a curvature ratio of 10 to 4, so this similarity of our results with those of Pui et al. is expected.
Although there is no apparent effect of Reynolds number for Re < 10,000, close inspection of Figure 3 reveals a notable difference as Re is increased and Stk is small. Where the statistically determined error bars for the same data point do not overlap between the different Reynolds number cases, significant differences are said to be detected. Specifically, at a Stokes number of 0.15, the Reynolds number cases 20,500 and 30,750 exhibit significantly more deposition than the 10,250 case. The absolute difference in deposition fraction from Re = 10,250 to Re = 30,750 for Stk = 0.15 is 0.22, which corresponds to a factor of 2.6 increase in deposition. Conversely, for larger Stk (Stk > 0.4), there is no obvious effect of Re on deposition fraction. Additional experiments were performed for a fixed Stk = 0.12, while the Reynolds number was varied from 5,000 to 30,750 and plotted in Figure 4. These results show quite significant variation in deposition fraction from 0.025 to 0.325 (or an order of magnitude increase) as Reynolds number is increased.
Since the Reynolds number effect is seen in the lower Stokes number range, it is possible that it is due to turbulent dispersion. The smaller Stokes number particles tend to be more responsive to the diffusive effect of turbulence. To more closely examine the complex effect of turbulence on particle trajectories, numerical simulations were performed, as discussed in the next section.

**Numerical Methodology**

The Reynolds Averaged Navier-Stokes equations (RANS) were solved with the CFX-11.0 software (ANSYS Inc.). To solve for the turbulent stresses in the RANS equations, the Shear Stress Transport (SST) turbulence model, developed by Menter (1994), was used. For the simulations, an entrance length of 10 inches (25.4 cm) and an exit length of 4.6 cm were also modeled. Figure 5 shows a schematic of the computational domain and an example of the cross-sectional mesh. The boundary conditions were: an area-average zero gauge pressure at the pipe outlet, a given flow rate of 22°C dry air, and a uniform velocity profile at the inlet to the entry length with an entrance turbulence intensity of 5%. The particles were introduced uniformly across the inlet of the entry length and were allowed to deposit in the entrance pipe before the bend. The number of particles deposited in the entrance pipe was subtracted from the number entering the inlet, so that the number entering the bend was known.
Figure 5: a) Computational domain and b) cross-sectional mesh for 1 150 000 nodes

Mesh

A multi-block structured grid with 15 blocks and a geometric accumulation of nodes toward the wall (closest nodes located at 0.06 mm away from the wall), was generated using the ICEM-CFD software (ANSYS Inc.). A grid convergence study using three grid sizes (640 000, 1 150 000, and 1 720 000 nodes) was performed for the flow-field for the Re = 30 750 case and results were evaluated using both the pressure drop through the test section (including the entrance pipe and exit pipe), and the velocity profile at both the start and the end of the bend as shown in Figure 6. The medium sized grid was considered to be adequate and the particle tracking simulations were performed with 1 150 000 nodes.
Figure 6: Grid convergence study of the velocity into and out of the bend ($Re = 30\,750$)

Figure 7: Grid convergence study of the secondary flow velocity component in the direction away from the bend center of curvature at the bend exit ($Re = 30\,750$)
Since the secondary flow pattern might have a significant effect on the deposition fraction results, a graph was also made of the velocity component pointing away from the center of curvature of the bend at the bend exit, shown in Figure 7. The secondary flow pattern was not significantly changed by increasing the node number from 640,000 to 1,720,000. Figure 8 shows the magnitude of mean velocities and the turbulence kinetic energy at several cross sections and at the midplane of the bend for Re = 30,750 and mesh size of 1,150,000 nodes. Magnitudes of the calculated turbulent kinetic energies were used in the analytical analysis presented in the discussion.

![Figure 8: Velocity magnitude and turbulence kinetic energy from RANS Results, Re = 30,750](image)

**Particle Trajectory Calculation**

Each particle trajectory is calculated by the software using the Lagrangian equation of motion for a spherical particle. The particle equation implemented in CFX11.0 (ANSYS Inc.) is
\[ m_p \frac{d\vec{V}_p}{dt} = \frac{1}{2} C_D \rho \vec{A}_p (\vec{V} - \vec{V}_p) (\vec{V} - \vec{V}_p), \]  

(4)

where \( m_p \) is the mass of the particle, \( \vec{V}_p \) is the particle velocity, \( C_D \) is the drag coefficient, \( A_p \) is the particle cross-sectional area, and the instantaneous fluid velocity at the particle location is given by

\[ \vec{V} = \vec{V}_m + \vec{V}', \]  

(5)

which is the sum of the mean velocity given by the RANS results, and a turbulent fluctuation, \( \vec{V}' \), which is modeled. The coefficient of drag is given by the Schiller-Naumann correlation (Schiller & Naumann 1935):

\[ C_D = \frac{24}{\text{Re}_p} (1 + 0.15 \text{Re}_p^{0.687}) \]  

(6)

For the instantaneous fluid velocity, \( \vec{V} \), in the particle equation of motion, there are two options available in the CFX11.0 software. \( \text{Re}_p = |\vec{V} - \vec{V}_p| d_p / \nu \) is the particle Reynolds number, where \( |\vec{V} - \vec{V}_p| \) is the modulus of the relative velocity between fluid and particle, \( d_p \) is the particle diameter, and \( \nu \) is the kinematic viscosity of the fluid. Option 1 is to use the mean flow calculated by the RANS solution. This omits the fluctuations in the fluid velocity resulting from turbulence, and so neglects the turbulent dispersion mechanism by making the fluctuation component equal to zero, \( \vec{V}' = 0 \), so that

\[ \vec{V} = \vec{V}_m. \]  

(7)

Option 2 considers turbulent dispersion, where \( \vec{V}' \) is generated through modeling using, for example, the eddy interaction model (EIM, Gosman, & Ioannides, 1981). Both approaches are tested in the current work. Deposition resulting from the mean flow tracking (option 1) is referred to as mean-flow induced inertial impaction. The effect of turbulent dispersion in addition to mean-flow induced inertial impaction is captured with the turbulent dispersion option (option 2). With the eddy interaction model, for each velocity component a turbulent fluctuation component is added to the mean (from the RANS results), by sampling from a Gaussian probability density function of zero mean and a standard deviation equal to the eddy
velocity scale, $u_c$.

$$u_c = \sqrt{\frac{2k}{3e}}$$ \hspace{1cm} (8)

This is the rms speed for isotropic turbulence. This fluctuation component is associated with a turbulent eddy, of a length scale, $L_c$,

$$L_c = 0.164 \frac{k^{3/2}}{\varepsilon}$$ \hspace{1cm} (9)

and is imposed for a certain interaction time after which a new fluctuation component is generated. Here, $\varepsilon$ is the rate of viscous dissipation. The interaction time is the minimum of the eddy time scale ($L_c/u_c$), or the time for the particle to traverse the eddy (given by the solution of the particle equation of motion). The turbulent dispersion model within CFX 11.0 is actually implemented as a dynamic hybrid between options 1 and 2 described above. To make the calculation more robust, the solver detects whether the flow is turbulent at the particle location (during the integration of the particle equation of motion) using the ratio between the eddy viscosity and the dynamic fluid viscosity. If this ratio exceeds a limit (typically 5), turbulent dispersion is used as described above. If the ratio is below the limit, then mean flow tracking (option 1) only is used. This hybrid turbulent and mean flow tracking approach was used in the present analysis when calculating turbulent dispersion results.

The particle density was fixed at a specific gravity of 0.912 (density ratio of 770). For a particular Stokes number (or at a fixed Re, for a given particle diameter) at least 20 000 particles were allowed to enter the bend. Since some particles deposited in the entrance pipe, more than 20 000 needed to be released into the start of the entrance pipe (with uniform spatial distribution) to ensure sufficient particles entered the bend. For simulations including turbulent dispersion, the deposition fraction in the inlet tube alone varied between 0.003 and 0.350 depending on the flow conditions. The released particles were tracked by the software using the particle equation of motion, with and without turbulent dispersion, and were used to calculate the deposition fraction. Particles impacting the bend walls were assumed to stick. Particles impacting the outlet tube were allowed to rebound (i.e. 100% coefficient of restitutions for both parallel and perpendicular directions from the walls).

An additional grid convergence analysis concerning deposition fraction for particles with a Stokes number of 0.07 at Re = 30 750 was performed, as shown in Figure 9. The Stk=0.07 case was considered the most challenging numerically when compared against results for higher Stokes numbers, which are not shown here. Twenty thousand particles were released in the simulations for three different grid sizes (640 000, 1 150 000, and 1 720 000), all using mean flow tracking. The deposition fraction seems to approach an asymptotic behavior from
1 150 000 to 1 720 000 nodes, indicating that the adopted resolution (1 150 000) is also adequate when particle deposition is concerned. Ten to thirty thousand particles were also tested with the 1 150 000 node mesh, which showed a negligible percentage variation (0.11%) in the deposition fraction, indicating that the number of particles released (20 000) was also adequate for particle deposition analysis.

![Figure 9: Grid convergence analysis for particle deposition calculations at Re = 30 750](image)

**Results and Discussion**

The numerical results for particle deposition fraction are plotted alongside the experimental data in Figures 10 and 11. Figure 10 shows simulation results using mean flow tracking only. Results with turbulent dispersion turned on are graphed in Figure 11. In Figure 10 it is evident that the numerical results without the turbulent dispersion (mean flow tracking) closely resemble the experimental results, except that there is no apparent Reynolds number effect at Stokes numbers near 0.15. However, the general agreement with the experimental data indicates that the particle deposition in the present geometry is dominated by a mean-flow induced inertial impaction mechanism. It is noted that for Stk>0.2, the RANS simulations show a slight decrease in deposition fraction with increasing Re. The differences are not noted in the experimental data (although the magnitude is small enough that it could be contained within the uncertainty bars), and it is conjectured that this is artifact of the intrinsic assumptions in the turbulence modeling for the RANS model of the fluid flow within the bend.
Figure 10: Comparison of numerical calculations of deposition using mean flow tracking with experimental results

Figure 11: Comparison of numerical calculations of deposition using turbulent dispersion tracking with experimental results
The absence of a Reynolds number effect in the mean flow tracking simulations for low Stokes numbers, suggests that the Reynolds number effect found in the experiments could be due to a turbulent dispersion deposition mechanism. However, in Figure 11 it can be seen that the numerical results with the inclusion of turbulent dispersion do not agree with the experimental results nearly as well as the mean flow tracking results in Figure 10. For Stokes numbers less than about 0.2, the numerical deposition results significantly overpredict the experimental results. This is not unexpected, since turbulent tracking is known to give considerable overprediction of particle deposition for small particles (Matida, 2000). At low Stokes numbers, the assumption of isotropic turbulence in the standard eddy interaction model overestimates the velocity fluctuations normal to the walls, leading to an overprediction of particle deposition. Experimental observations indicate that the stream-wise turbulence kinetic energy component is approximately double the cross-stream component and about 4 times the magnitude of the wall-normal component in the boundary layer region close to the wall (see Kundu & Cohen, 2002, as a general reference). Away from the wall toward the edge of the boundary layer, the turbulence intensity tends to become isotropic.

While a Reynolds number effect is apparent in the numerical results, it is present for all Stokes numbers in contrast to the experimental results, which showed Reynolds number dependence only for Stokes numbers smaller than approximately 0.4. Higher Reynolds numbers produced higher deposition fractions but curiously, for Stokes numbers greater than 0.6, the inclusion of turbulent dispersion actually reduced the deposition fraction for the Re = 10 250 case and increased it for the Re = 30 750 case (compared to mean flow tracking). It should be pointed out that the simulation of the single phase flow using RANS equations and an isotropic eddy (turbulent) viscosity does not account for curvature effects (neither wall nor streamline curvatures), as discussed by Davidson (1995), and the present deposition results should be used with some caution. However, it is conjectured here that the differences between the mean flow tracking curve (impaction without dispersion) and the experimental data are the result of enhanced deposition by turbulent dispersion from coherent structure within the turbulence that is not correctly captured by simulations. This hypothesis is discussed further in a simple analytical analysis presented in the next section.

It should be noted that the turbulent tracking deposition in the bend is also affected by the turbulent dispersion of particles in the inlet pipe (prior to the bend). Eddy interaction models are known to cause a “spurious” concentration of particles in regions of low turbulence kinetic energy (near the walls in the present case) for particles having behaviour approaching that of fluid tracers (MacInnes and Bracco, 1992). However, this was not apparent in the present simulations and the particle deposition pattern instead showed expected concentrations of particles along the outside of the bend, with the inside of the bend being relatively free of deposition. This is the same pattern seen by Breuer et al. (2006) in LES simulations for a Reynolds number of 10 000, for which the concentration towards the outside of the bend becomes more pronounced as the Stokes number increases, and there is a strip on the inside of the bend free of deposition.
Analysis

Even though the turbulent tracking results carry some modeling uncertainty, the flow-field from the RANS calculation can still be examined to support an analytical estimate of particle response to turbulence. Figure 12 shows the turbulence intensity \((I)\), half-way around the bend and at the bend exit. The normalized eddy velocity scale \((u_e^{(n)})\) is

\[
u_e^{(n)} = \sqrt{\frac{2k}{\nu}}.
\] (10)

The turbulence intensity \((I)\) based on the mean flow speed is (Schlichting, 1968)

\[
I = \sqrt{\frac{u_e^2 + v_e^2 + w_e^2}{V}} = \sqrt{\frac{2k}{V}} = \sqrt{3u_e^{(n)}},
\] (11)

for isotropic turbulence.

Figure 12: Turbulence intensity \((I)\) half-way around the bend and at the exit plane
In Figure 12, it can be seen that the turbulence intensity is maximum close to the outside wall, but drops off to zero at the walls. The magnitudes are similar among the 3 cases. As the flow pattern develops with distance around the bend, this distribution becomes more skewed toward the outside of the bend. The impact of turbulence on the EIM occurs through the turbulence intensity (Equation 11) and the frequency of interaction with successive eddies (eddy interaction time). The impact of these on the particle equation of motion determines the modeling effect of the turbulence.

The Stokes drag law can be used \( C_D = \frac{24}{Re_p} \) for making a simplified analysis of the response of particles to turbulent fluctuations (Tavoularis, 2005). The simplified particle equation of motion (with velocities normalized by the average fluid velocity \( V \)) is then (see Wilson, 2008):

\[
\frac{aStk}{V} \frac{dV_p^{(n)}}{dt} + V_p^{(n)} = \bar{V}^{(n)},
\]

where \( V_p^{(n)} \) and \( \bar{V}^{(n)} \) are in turn, the normalized particle velocity and the normalized fluid velocity. Breaking the fluid into mean and fluctuating components \( \bar{V} = \bar{V}_m + \nu \) gives:

\[
\frac{aStk}{V} \frac{dV_p^{(n)}}{dt} + V_p^{(n)} = \bar{V}_m^{(n)} + \nu^{(n)}. \tag{13}
\]

This is a first order ordinary differential equation, with two inputs, the mean flow and the fluctuating flow. The solution is a sum of the solution to the mean flow and the solution to the fluctuating flow. We are interested in the solution to the fluctuating flow, so we want to solve

\[
\frac{aStk}{V} \frac{dV_p^{(n)}}{dt} + V_p^{(n)} = \nu^{(n)}. \tag{14}
\]

The fluctuating component is composed of the range of frequencies making up the frequency spectrum of the turbulence. For a first order system with a sinusoidal input, the amplitude ratio \( (\eta) \) is

\[
\eta = \frac{1}{\sqrt{1 + (\omega \tau)^2}}, \tag{15}
\]

where \( \omega \) is the input circular frequency and \( \tau \) is the time constant. In this case, the time constant is

21
\[ \tau = \frac{aStk}{V}, \]  

so the ratio of the amplitudes of the particle fluctuations to fluid fluctuations is

\[ \eta = \frac{1}{\sqrt{1 + \left( \frac{\omega aStk}{V} \right)^2}}. \]  

This amplitude ratio is a quantitative indication of the ability of a particle to track turbulent fluctuations in the flow occurring at a specified frequency. An amplitude ratio near unity indicates that the particle and flow fluctuations are similar, which means that the particle is able to follow turbulent fluctuations at the specified frequency. Conversely, a small amplitude ratio means that flow fluctuations are damped by the particle inertia so that in effect, the turbulent flow fluctuations are filtered out, and the particle does not respond to the relevant frequency of fluctuation.

To examine the source of the Re effect on deposition efficiency seen in the experiments, the amplitude ratio can be analyzed at the eddy frequency scale (the overall average frequency used for the EIM), where

\[ \omega_e = \frac{2\pi}{0.20} \frac{k}{\varepsilon} \]  

and the denominator is the eddy time scale. The eddy frequency equals 2.83 times the turbulent eddy frequency in the SST turbulence model, which was used in the RANS calculation. If one hypothesizes that the observed Re effect can be attributed varied amounts of filtering of turbulent frequencies due to particle inertia, then an effect should be apparent in the calculated amplitude ratio of the particle response.

In addition to the eddy frequency scale, particle response can also be calculated for the frequency associated with the largest eddies, which is roughly given by

\[ \omega_k = \frac{2\pi}{a/u_e} = \frac{2\pi}{a} \sqrt{\frac{2k}{3}} \]  

In this case, the time scale used is simply the pipe radius (a=0.0051 m) divided by the eddy fluctuation velocity scale.

To obtain a value for the turbulent eddy dissipation, \( \varepsilon \), and turbulent kinetic energy, \( k \), volume averages from the RANS results were used (averaged over the domain within the bend) at the three different Reynolds numbers. These values are summarized in Table 2 where the kinematic viscosity for air is assumed to be \( \nu = 1.53 \times 10^{-5} \text{ m}^2/\text{s} \). Table 2 also shows the frequency scales and corresponding amplitude ratios for particle response as a function of Reynolds number.
Table 2: Turbulent frequency scales and corresponding particle amplitude ratios

<table>
<thead>
<tr>
<th>Re</th>
<th>V  [m/s]</th>
<th>$\varepsilon$ [m²/s³]</th>
<th>$k$ [m²/s²]</th>
<th>$\omega_e$ [$10^4$ s⁻¹]</th>
<th>$\omega_L$ [$10^4$ s⁻¹]</th>
<th>$\eta_e$</th>
<th>$\eta_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 250</td>
<td>15.4</td>
<td>3442</td>
<td>2.128</td>
<td>5.1</td>
<td>0.15</td>
<td>\frac{1}{\sqrt{1+284Stk^2}}</td>
<td>\frac{1}{\sqrt{1+0.1332Stk^2}}</td>
</tr>
<tr>
<td>20 500</td>
<td>30.8</td>
<td>23071</td>
<td>7.968</td>
<td>9.1</td>
<td>0.28</td>
<td>\frac{1}{\sqrt{1+228Stk^2}}</td>
<td>\frac{1}{\sqrt{1+0.1332Stk^2}}</td>
</tr>
<tr>
<td>30 750</td>
<td>46.2</td>
<td>65848</td>
<td>16.32</td>
<td>12.7</td>
<td>0.41</td>
<td>\frac{1}{\sqrt{1+197Stk^2}}</td>
<td>\frac{1}{\sqrt{1+0.1334Stk^2}}</td>
</tr>
</tbody>
</table>

The amplitude ratios of the particle and flow fluctuations are graphed in Figure 13, for different frequency scales and Reynolds numbers to show the calculated particle amplitude response to the turbulence. From Figure 13, it is evident that the particle amplitude response to the lowest frequencies in the turbulence is approximately 100% for all three Reynolds numbers, and for Stokes numbers below 1. However, there is significant damping for fluctuations occurring at the much higher eddy frequency scale, where the inertia of the particles prevents tracking of the smaller scales in the flow. For Stokes numbers larger than approximately 0.4, the amplitude response of the particle is almost negligible at the eddy frequency scale ($\eta<0.2$) for all three Reynolds numbers. For Stokes numbers less than approximately 0.4, the particle response to the eddy frequency scale is more significant, and increases strongly as the particle Stokes number decreases.
Based on the results shown in Figure 13, it can be generally said that high frequency eddies that are added to the frequency spectrum of the turbulence with increased Reynolds number will not be felt by particles with Stokes numbers greater than approximately 0.4. Conversely, for particles with Stokes numbers below 0.4, the increase in the range of the frequency spectrum due to an increase in the Reynolds number will affect the particle response.

More interesting is the apparent influence of Reynolds number on the particle response to the eddy frequency scale. Although the response curves for the three Reynolds numbers are quite similar, for fixed Stokes numbers near 0.1, the curves trend more vertically and there are significant differences in the damping levels from Re = 10 250 to Re = 30 750. To illustrate this more clearly, three additional curves are plotted on the right axis of Figure 13 showing the difference in amplitude response for different Re. These curves confirm that the model predicts a peak effect due to increasing Re at Stk ≈ 0.1 and a negligible effect for Stk > 0.4, which matches the trends seen in the experimental data. Thus, the ability of particles at low Stokes numbers to follow turbulent eddies would vary measurably for Re from 10 250 to 30 750, and would noticeably affect turbulent dispersion and deposition for a narrow range of particles with 0.1 ≤ Stk ≤ 0.4. Close inspection of the analytical results also suggests that the response curves at the eddy frequency scale trend together at higher Re (i.e. more separation between Re = 10 250 and 20 500 than Re = 20 500 and 30 750). This effect was also apparent in the experimental results. Thus, although the analytical model is quite simplified in that only
isolated scales are considered, it does potentially explain the presence of an 
Re effect on turbulent deposition in bends shown by the present experiments that to the authors’ knowledge has not previously been seen in the literature.

Conclusions

Particle deposition in a 90° bend with a curvature ratio of $R_b/a = 7.4$ was studied experimentally and numerically for a range of Reynolds numbers ($Re = 10\,250, 20\,500, 30\,750$) based on the inlet diameter that were higher than most previous studies where $Re \leq 10\,000$. The Stokes number was varied from 0.1 to 1.0. The overall experimental results indicate that there is a notable Reynolds number effect on the deposition fraction for turbulent bend flows at low Stokes numbers. For Stokes numbers of 0.12, the measured deposition fraction increases by a factor of six (from approximately 0.05 to 0.31) as the Reynolds number is increased from 10\,250 to 30\,750. Numerical calculations using the RANS equations with the Shear Stress Transport turbulence model were performed to see whether the experimental result could be better understood. There was good agreement between the numerical results and the experimental results for mean flow tracking, indicating the dominant influence on particle deposition is by mean-flow induced inertial impaction. However, the RANS simulations with an eddy interaction model for particle dispersion by turbulence were not able to accurately capture the additional deposition for increased $Re$ that was measured experimentally at low Stokes numbers. From an analytical analysis of the flow-field based on turbulence data from the RANS results, particle response equations were calculated for different turbulent frequencies found in the flow as a function of $Re$. Based on the calculated damped inertial response of particles to turbulence in the flow, this analysis suggests that turbulent dispersion would lead to increased deposition for $Stk \sim 0.4$ over the range of $Re$ considered, as was demonstrated experimentally. The analytical calculations at the eddy frequency scale show a non-linear influence of $Re$ on particle response, and correctly predicted a peak effect for $Stk \approx 0.1$, consistent with the experimental results. In theory, more detailed simulations using Large Eddy Simulation or Direct Numerical Simulation should be able to capture this effect.
References


